Chapter 2

Linking Psychometric and Cognitive-Developmental Frameworks for Thinking About Intellectual Functioning

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Abstract

Two major approaches to understanding intellectual functioning – the psychometric and the cognitive-developmental approaches epitomised by the seminal work of Binet and Piaget, respectively – are here considered complementary rather than incommensurable and, in particular, as essentially manifestations of the same underlying construct but at different levels of scale. From this perspective, and by exploiting Item Response Theory, performances of persons on *Raven's Progressive Matrices* (exemplifying the psychometric approach) and performances on three Piagetian tasks (the Balance, Chemical Combinations, and Correlations tasks) are mapped onto a single continuum of intellectual development. *The implication is that qualitative and quantitative conceptions of intellectual development are closely interlinked: within each cognitive-developmental stage, a series of small, incremental, quantitative changes occur and evolve into a major qualitative change in cognitive functioning.* In order to clarify the nature of the transformations in thinking that occur at the transition points between one Piagetian stage and another new taxonomy is developed to classify RPM items. Knowledge of the RPM items which are operational at each Piagetian transition indicates the transformations in thinking that are required.

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This paper is divided into two Parts. Part I describes the data base for both studies and outlines the way in which both the difficulty of Raven Progressive Matrices (RPM) items and developmental levels in thinking as revealed by interviews carried out using three Piagetian tasks were mapped onto a common continuum … and the conclusions to be drawn from the demonstration that it is, indeed, possible to do so. Part II develops a new taxonomy for classifying RPM items and examines those that are operational at each Piagetian transition point.

PART I

The development and expression of intellectual functioning has been, and is, of major interest to psychologists and educationalists. Although a range of different perspectives for studying intellectual development have been put forward, two major perspectives dominate: one, which has developed out of the work of Binet, is often termed the *psychometric approach*; the other, which has developed out of the work of Piaget, is often termed the *cognitive* or *stage-developmental* approach.

According to the psychometric perspective, intelligence is conceptualised as a set of quantifiable dimensions along which people can be ordered (Seigler & Richards, 1982) and along which people may advance as growth takes place. According to the cognitive or stage-developmental perspective, people pass through four major stages identified by qualitatively different modes of reasoning which are universal and irreversible and which transcend substantive domains, although the rate of development may depend on individual differences, including differences in experience (Piaget, 1972). Development through the stages is not discontinuous, but within each stage a person may reason in a relatively stable way for some time before moving relatively rapidly into the next stage (Case, 1978).

Many tasks have been developed to operationalize both these perspectives. In the psychometric approach, a set of relatively homogenous tasks relevant to a particular dimension of intelligence and of increasing difficulty, are constructed and the number of correct responses a person gives constitutes his/her location on the dimension. In the stage-developmental approach, sets of questions relating to a variety of tasks (usually involving concrete materials) are used to elicit the kind of reasoning persons employ to solve a given problem and, on the basis of the nature of their reasoning, persons are deemed to be operating at Stage I (pre-operational), Stages IIA or IIB (concrete operational, A or B), or Stages IIIA or IIIB (formal operational A or B) (Inhelder & Piaget, 1958). The quality of reasoning, rather than the "correctness" of the solution, is the relevant information which is assessed.

These two approaches have been viewed, mostly, as mutually exclusive, although some researchers have considered them complementary (Elkind, 1974; Flavell,1971). The latter view is taken in this paper. Historically, this complementarity seems to fit well with the fact that Piaget worked with Binet and began to develop his theory as a result of focussing on the errors children tended to make when attempting the Binet items and noticing consistencies amongst those errors. Substantively, similarities between the two include a view of intelligence as having an adaptive function, as changing in some way with age, and as becoming more complex and stable over time. The common charge (e.g., Farnham-Diggory, 1972) that a psychometric approach focuses exclusively on an outcome or product whereas a cognitive-developmental approach is concerned with the processes involved in thinking about and solving a task, seems too simplistic: the assumption of particular processes required to solve test items correctly is implicit (and is often explicit) in a psychometric approach (for example, Spearman postulated eduction of relations and correlates as being the fundamental processes associated with general intelligence) and, conversely, there are outcomes in Piagetian tasks which are judged as more or less successful at particular levels relative to an expected, correct outcome. The difference seems to be one of the focus of the task presented and, therefore, the level or scale of the assessment of the processing, rather than a different perspective altogether.

Methodologically, there are also similarities between the two approaches: for instance, from a psychometric perspective, Binet developed items that were judged appropriate for particular age levels (children of a specific age were expected, in general, to be able to complete these particular items successfully) and which were ordered in difficulty, that is, the items can be conceived as being on a continuum of increasing difficulty. From a cognitive-developmental perspective, children of a particular age are expected to be able to reason at a level generally characteristic of that age. Further, the notion of stages in development implies an order, a direction and a hierarchy of difficulty of tasks, or levels within tasks, that can also be conceived as being on a continuum. "Cognitive stages have a sequential property, that is, they appear in a fixed order of succession" (Piaget, 1970) and "each one of these periods or subperiods is necessary to the constitution of its successor" (Piaget, 1970) – any one stage is both an extension of the one before and the basis of the one following (Piaget, 1970). For these reasons, the notion of integrating psychometric and stage-developmental approaches to intellectual development is theoretically consistent.

Previous research has attempted to investigate the relationship between psychometric measures of intelligence and Piagetian measures using factor analysis. Earlier conclusions from this research were that the two measures were distinct (de Vries, 1974; de Vries & Kohlberg, 1977) and that they, therefore, addressed different types of intellectual functioning (although some of the research was questioned on the grounds of faulty methodology (Carroll, Kohlberg & de Vries, 1984)). A more recent attempt at explicating the relationship between the two approaches (Lim, 1988) concluded that they both contributed to a ***g***, or general intelligence factor and that, in addition, there was a distinct formal operational factor as well as spatial, numerical and verbal factors. However, a major weakness of factor analysis as a methodology is that the correlations reflect the relationship between the relative difficulty levels of the items and the location of the abilities of the people relative to these items rather than the structure of the variables (Duncan, 1985; Styles & Andrich, 1994). Factors may then be, at best, no more than difficulty factors.

Mathematical Modelling of Intellectual Development

Little work has been done in applying formal mathematical modelling to the study of aspects of Piaget's theory, although three exceptions have been Andrich and Constable (1984), Bond (1993), and Davison, King, Kitchener, and Parker (1980).

> "Clearly, the sequencing of stages implies an ordered and hierarchical progressing, but there are two mechanisms which may satisfy such a structure. One is the *cumulative* mechanism most simply understood as exhibiting a Guttman scale. According to this mechanism, if one can reason at a given stage, then one can reason also at earlier stages. The other is the *unfolding* mechanism, most simply understood in terms of Coombs' work in choice data. According to this mechanism, if a person is to choose among a set of entities which have been ordered according to some principle, then the person will tend to choose the ones nearest to his or her ideal point and

tend to reject ones at greater distances from the ideal point, irrespective of direction. Analogously, a person will tend to apply a stage of reasoning corresponding to his/her present stage, and tend not to apply reasoning at either an earlier or a later stage" (Andrich & Constable,1984)

In the case of Piagetian tasks (as is the case in most psychometric tasks), the mechanism of responding to items is cumulative in that if a person is reasoning at, say, Stage IIB, then that person is deemed to have already passed through the lower stages I and IIA and not yet reached Stages IIIA or IIIB. Thus, although *development* is seen as unfolding in that a person at, say Stage IIB will not reason at lower level, the *measurement* of that development is cumulative.

In addition to articulating a mechanism which can be modelled, it is important to conceptualise the response mechanism as probabilistic, rather than deterministic. In a probabilistic model, a particular person will have a higher probability of responding in a mode appropriate to a certain stage than s/he would have of responding at a stage higher or lower, but some deviations from the expected stage (that is, the stage of highest probability) are consistent with this conceptualisation. Thus it is not expected that the manifested and observed reasoning will accord perfectly and be recorded unequivocally as being at a particular stage, as it is in a deterministic model. This probabilistic formalisation is consistent, for example, with Chomsky's (1976) conceptualisation of the difference between competence and performance, in that a number of influences (such as anxiety or a lapse of concentration) may cause a person to reason at a lower stage (level of performance) than that person is actually capable of (level of competence).

From this perspective, it should be possible (as is the case with psychometric test items) to order cognitive-developmental transition points between levels of reasoning on different tasks along a continuum of difficulty. The idea of such an ordering of cognitive tasks involving qualitative data was first formulated by Thurstone (1925). For intelligence, aptitude or achievement tests, he considered the possibility of mapping the levels of difficulty of tasks onto a continuum according to their relative difficulties.

We shall, therefore, locate these test questions on the scale as landmarks of different levels of intellectual growth (Thurstone, 1925).

In the same way, attitude statements could be characterised as landmarks on an affective continuum according to their relative affective values. Thurstone's work provides a conceptualisation for reconciling psychometric and stage-developmental theories which was elaborated by Andrich and Constable (1984), using Rasch (1960/1980) models for measurement. In particular, one of these probabilistic mathematical models – the Extended Logistic Model (ELM) – provides a method for attaining this theoretical reconciliation by allowing the simultaneous mapping of dichotomous and polychotomous items (Andrich,1985). Dichotomous task (item) difficulties located on a continuum can be considered as single transition points or thresholds *between* tasks, whereas Piagetian tasks have more (polychotomous) responses with transition points *within* tasks. Note again that the response mechanism underlying the model is a probabilistic rather than a deterministic one. If it can be shown that the data of interest fits the model, then that data can be considered, at one level, to form a continuous, unidimensional scale. The methodology provides the possibility of investigating whether the data conform to the model and, therefore, whether the theoretical reconciliation is tenable.

Method

Research Design

A combination of longitudinal and cross-sectional designs was used to examine within and between cohort differences in intellectual development over a period of 6 years. Initially, three age cohorts were tested (10, 12, and 14 years) with roughly equal numbers of children (60) in each group and each group consisted of approximately equal numbers of boys and girls. Mean ages of the three groups at the first test occasion were 10.03, 12.09 and 14.07 years. All the children came from two Perth (Western Australia) metropolitan schools and from medium to high socio-economic status families. Children were matched on birthdate across the three groups for the two sex groups separately. Each child was tested at six monthly intervals on the psychometric variable (the RPM). After the first test occasion, a subset of 60 children was selected – equal numbers from each age and sex group – to be tested on the stage-developmental variable (three Piagetian tasks) at yearly intervals. These children were selected to be representative of the ability range of the entire sample of children as shown by performances on the psychometric variable on the first occasion.

Parental permission to participate was obtained for each child and only one child was excluded because parental permission was not given. The attrition rate was small: under 6% over six years. Children were tested a maximum of 10 times on the psychometric variable and four times on the stage-developmental variable. They were not tested beyond age 16, hence there was a limited amount of data collected on the children who were initially 14 years old.

The Psychometric Variable – Raven's Progressive Matrices

The variable chosen to study intellectual development from a psychometric perspective was operationalised by the *Raven's Progressive Matrices* (RPM). Raven described his test as "a test of person's present capacity to form comparisons, reason by analogy and develop a logical method of thinking regardless of previously acquired information" (Raven, in Burke, 1958). It was originally developed to assess one of Spearman's (1927) two components of "**g**" or general intelligence – that of eductive ability (Raven, J. C., 1940; Raven, J., 1989), the other being reproductive ability. This has been supported by the results of Snow, Kyllonen and Marshalek's (1984) study which showed the RPM to measure abilities central to the concept of general intelligence.

The many advantages of this test, the reasons it was chosen and the way it was computerised and administered, has been described in detail in Styles and Andrich (1993).

There are four forms of the Matrices: the Coloured (suitable for persons less than 10 years of age), the Standard (for persons from about 10 to 15 years of age) and Advanced Sets I and II (for adults or able younger persons). All forms except the Coloured form were used in this study. The initial item in each of the five sets of items in the Standard form were used as examples. All the rest (103 items in all) were ordered in difficulty and administered individually in computerised format. Details of the administrative procedures can be found in Styles (1991).

The Stage-Developmental Variable – Three Piagetian Tasks

The variable chosen to define intellectual development from a stage developmental approach and, in particular, to focus on formal operational thinking, was operationalised by performance on three Piagetian tasks: Equilibrium in the balance, Chemical combinations and the Correlations task. Inhelder and Piaget (1958) have discussed the transformation of modes of thought pertinent to the emergence of formal operational thinking. These operations include conservation, combinatorial operations, the notions of inversion, reciprocity and proportionality, and correlation. In this study, tasks to provide evidence of the level of chidren's thought processes involving all but the first-mentioned operation (conservation) were used. Piaget & Inhelder's discussion seems to indicate these operations are ordered at one level (Inhelder & Piaget, 1958), therefore it is possible to conceive of three of these operations (the proportional, combinatorial and correlational operations) as being of increasing difficulty and, in fact, requiring increasing competence, with competence in the earlier ones being necessary for competence in the last. In particular, the combinatorial operation seems to be regarded as fundamental in the development of formal operations. "The combinatorial operations do not actually belong to the set of propositional operations and do not derive from them; on the contrary, they are the prerequisite conditions of their development (Inhelder & Piaget, 1958). The proportionality schema is seen as effecting "the transition between schemata" (Inhelder & Piaget, 1958) and is "inherent in the integrated structure which seems to dominate the acquisitions specific to the level of formal operations" (Inhelder & Piaget, 1958). And, finally, correlation is related to the concept of proportions and "depends on the propositional combinatorial system (Inhelder & Piaget, 1958). Thus, all three operations appear to be crucial in the development of formal operations and it was deemed important to include tasks designed to elicit them in the study. Although all of them are necessary in attaining the level of formal operations, as is the case with all Piagetian tasks, stages of development within each task toward full use of these operations can be identified and have been characterised accordingly (Inhelder & Piaget, 1958).

The three specific tasks chosen were (1) Combinations of Coloured & Colourless Chemical Bodies (Chem); (2) Equilibration in the Balance (Bal); and (3) Correlations (Corr). Inhelder & Piaget (1958) provide a detailed description of these tasks.

Administration

All three tasks were administered in the same order (Chem, Bal and Corr) once a year for three years and the last occasion (fourth) occurred after eight months, because a group of children were about to leave school.

On occasions two to four, the Chemical Combinations task was replaced by the electrical equivalent of the task (Philp & Kelly, 1974) because the administration was quicker and less messy and these were important factors to consider when the children could spend a limited amount of time away from their classes.

Questions were presented in a semi-structured format in that a basic set of questions was presented, but the administrator used additional probing questions when necessary. All children were given pencil and paper to use if they wished. Administrators were not aware of the level of the child's intellectual performance on the Matrices or in academic areas, except for one administrator on the first occasion who was aware of the children's general level of performance on the RPM. All interviews were recorded on tape and assessed by two judges according to coding schedules developed from the characteristics of each stage of reasoning as described in Inhelder and Piaget (1958).

Coders were trained in assessing the interviews using a few interviews which could not be used because the respondents had left the schools. The coding of the first three sets of data were checked for reliability by a second coder. The inter-judge reliability was 73%. The coding for a subset of the last set was checked by a second coder and any discrepancies arising by either method were resolved by discussion between the two coders.

Data Analysis

All responses to the RPM and the three Piagetian tasks were analysed according to the Extended Logistic Model and mapped onto the same continuum. Specifically, the location of the items of the RPM and the thresholds marking off the transition points between the stages of the cognitive developmental tasks were located on the same continuum. The procedure used to analyse the data involved a pair-wise estimation algorithm, often employed by Choppin (1968, 1983), which accounts routinely for missing data and therefore permitted a joint analysis of data where more children had responded to the RPM than to the Piagetian tasks and where not all the RPM items were attempted by everyone on all occasions. The computer program used was ASCORE (Andrich, Lyne, & Sheridan, 1990), which also provides statistical tests of fit enabling a check of the conformity between the data and the model.

Results

When the psychometric (103 RPM items) and the stage-developmental (3 Piagetian tasks) variables were scaled jointly using the unidimensional Extended Logistic Model, the fit to the model was satisfactory. These results mean that the psychometric and stage-developmental variables formed a unidimensional scale on a single latent trait at the level of precision provided by the data. From these results, the interpretation is that, at one level, the reasoning processes underlying both types of variable are similar: the underlying latent trait is postulated to be that of abstract reasoning – considered by Elkind and others to be the fundamental reasoning process in intelligent thought (Elkind, 1974).

Because the item difficulties and person ability estimates are in the same metric, and given that the items fit the model, the difficulties of all the items relative to each other can be examined. These are depicted in Figure 2.1 along with the frequency distribution of the persons.

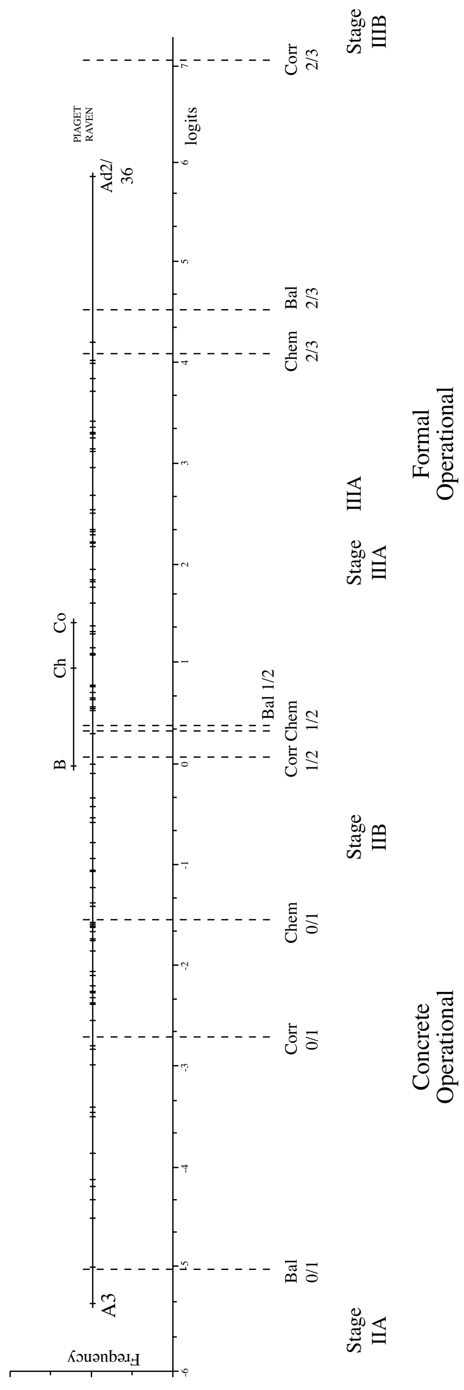
As can be seen, the Piagetian items are of different difficulties from one another and are ordered as expected with the proportionality (Bal) task being the easiest and the correlational task (Corr) being the most difficult. Relative to the spread of difficulties in the RPM, they are fairly similar to one another in average difficulty and correspond to the middle of the range locations of items on the RPM.

Because the transition points between stages of the cognitive developmental tasks are in the same metric as the locations of the items, it is possible to plot the category characteristic curves for the three Piagetian tasks on the same scale and then examine where the transition points occur relative to the RPM item locations (as well as the ability estimates). These curves, which can be superimposed on the scale of item locations, are shown in Figures 2.2a, 2.2b, & 2.2c for the Bal, Chem and Corr tasks, respectively. Note that the transition points between stages are at different locations on the continuum for the three tasks and that the transition points between stages IIA/IIB and between stages IIIA/IIIB are much more varied, relative to one another, than are those between stages IIB and IIIA which are at virtually the same location for each of the three tasks.

Discussion

This study demonstrates the feasibility and usefulness of integrating two basic approaches to the study of the development of intellectual functioning. Using Rasch's extended logistic model, it is possible to quantify variables from the two perspectives and to scale them jointly onto a continuum that has a consistent unit of measurement throughout the operating range of the combined variables, enabling a direct comparison of the two types of items (tasks) and person performances on them. This, in turn, helps illuminate our understanding of the variables being examined: in this

Figure 2.1. **Joint Scaling of RPM and Three Piagetian Tasks**



case, the results indicate the two variables are different manifestations of the same latent trait which is postulated to be that of abstract reasoning. This supports the notion of Snow, Kyllonen, and Marshalek (1984) of the centrality of the RPM as a measure of general intellectual functioning. It also supports a similar notion concerning the centrality of Piagetian tasks as measures of the same construct.

The fact that, in the RPM, the earlier perceptual items scale together with the later more "analytic" ones also indicates that there is

Figure 2.2a. **Category Characteristic Curves for Chemical Task**

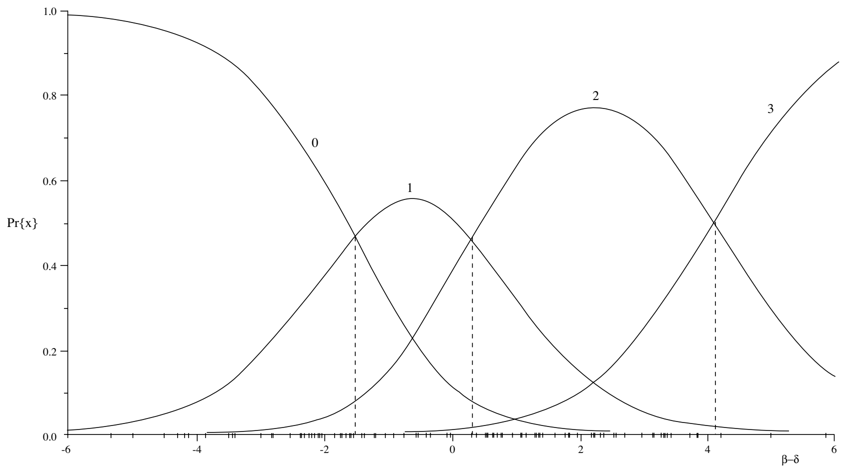


Figure 2.2b. **Category Characteristic Curves for Balance Task**

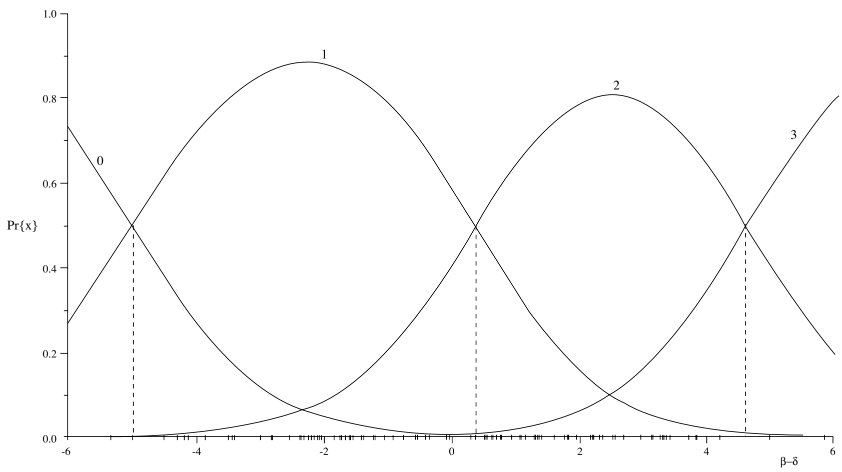
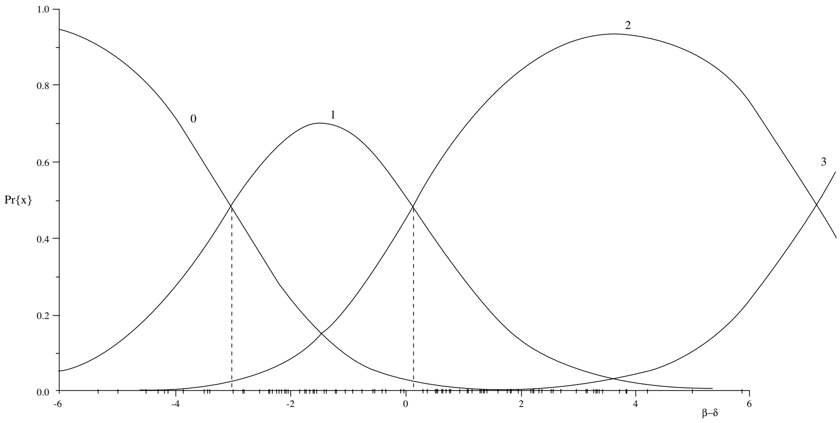


Figure2.2c. **Category Characteristic Curves for Correlation Task**



something common to both those types of processing, that is, the one type (perceptual processing) is a prerequisite for being able to use the other (analytic processing), and *they are both manifestations of a similar latent trait – that of abstract reasoning.* This conceptualisation fits well with the Piagetian sequence of stages in which preoperational thought (in which the person tends to interpret the world in terms of perceptual information) predates, and is a prerequisite for, operational thought in which the person is not at the mercy of purely perceptual information but is able to reason according to what the person *knows* to be the case rather than what s/he *sees* is the case.

Examining the relationships amongst the two operationalised variables of interest, it can be seen that they cover a wide – and a similar – range of ability, and that the top of the RPM coincides fairly well with the attainment of the second stage of formal reasoning (Stage IIIB) in two of the three tasks which were used. (The attainment of formal operations in the third task – Correlations – occurs beyond the location of the most difficult of the RPM items.)

The results show that these Piagetian tasks (and presumably others) are of different difficulties. The category characteristic curves indicate the probabilistic nature of stage categorisation and also show the possibility of stages for different tasks not being identical in meaning so that operating at a particular stage according to one task does not necessarily indicate the ability to do so at the same stage on another task for any one person. Considering this in more detail, it is evident from Figure 2.1 that although the transition points for the three Piagetian tasks occur at different positions on the continuum, this is so only for the transition points between sub-stages – the transition points for all three tasks between stage II and stage III are very close together. This indicates, firstly, that people develop the reasoning processes required for different tasks at different times, and thus some tasks are more difficult at that stage than others are. For example, it is more difficult to pass from IIA to IIB in the Chem than in the Bal task – at one level of scale, the processes required for the Bal task at this stage would seem to be acquired earlier than those for the Chem task. This is consistent with Flavell's explanation of asynchrony in development on different tasks being due to tasks requiring different levels of functional maturity (Flavell, 1971). Secondly, the close proximity of the IIB/IIIA thresholds for all three tasks suggests that there is a more significant, major transition point between stages II and III: it seems people need to develop thinking processes that are common to all the tasks before they can change from stage II to stage III. This supports Piaget's theory (Piaget, 1972) that people may exhibit discontinuity in developing reasoning processes for different tasks: however, despite this, the transition between stages II and III (concrete to formal operations) remains a major qualitative change that is common to many tasks. This interpretation also supports the contention of Fischer, Pipp, and Bullock (in Kitchener, Lynch, Fischer, & Wood (1993) in addressing the development of reflective judgment, that when a new developmental level is emerging, people spurt in a whole range of domains. If this is so, then development would appear more stage-like at major transition points than between these points when different skills are developing at different times and at different rates (that is, people might be spurting in a specific area at different times). It would seem, then, that with regard to a specific task, development would appear more stage-like (in that minor spurts would be more obvious) than if development is considered at a more general level when spurts in different tasks coincide to a greater extent. This means that the degree to which stage-like development is obvious will depend on the level of scale at which measurements are made, and the relationship of the scale to the tasks one is using.

In regard to the expected ordering of the three tasks (Chem, Bal, Corr), it would appear that, although this order may vary through the development of concrete operations, for the firm establishment of formal operations (the transition from IIIA to IIIB), the processes required by Chem (full understanding of the combinatorial – Stage IIIB) need to be in place before those for Bal are fully established (negation and reciprocity) and, similarly, those for Bal need to be in place before Corr is fully established. Thus, for the full attainment of formal operational thinking (stage IIIB), the order of establishment of reasoning processes seems to be in the order indicated by Inhelder and Piaget (1958), and Piaget's contention that the combinatorial underpins formal operational, propositional reasoning is supported.

Overall, the mapping of these two different variables – one from the quantitative, psychometric tradition and one from the qualitative, cognitive-developmental tradition, indicates that, at one level, quantitative and qualitative change are closely interlinked – the one *is* the other. Small quantitative increments eventually result in a major qualitative transformation of thought processes.

It is useful here to use an analogy to an evolutionary, biological process. Certain anatomical structures in early insect forms have the function of heat-regulation. However, above a critical body/wing ratio, these structures function, rather, as wings. Thus, more of the same results, not in a simple quantitative accumulation, but in a "complex reordering of parts with invention of new items" (Gould,1991). With respect to intellectual development, similarly, a sequence of simple and small quantitative changes translates into a major alteration of quality.

PART II

This Part of our paper seeks to extend the work summarized above by identifying parallels between psychometric and cognitive-developmental approaches and thus investigate the cognitive processing involved in solving the RPM items which correspond to the transition from concrete to formal operations on the Piagetian tasks.

A Taxonomy of Matrix Items

Central to the study is the characterisation of the algorithms or rules that a person needs to educe in order to solve RPM items. Previous work in the area includes that of Green & Kluever (1991), Hornke & Habon (1986) and Carpenter, Just, & Shell (1990). However, because these characterisations seemed to have disadvantages, this paper embodies an approach which differs from the others in several respects.

There have been several attempts at categorising matrix-like test items (particularly, the items of the different forms of the Raven Progressive

Matrices) either in terms of the elements of the items (the structure) (e.g. Green & Kluever, 1991, identified structural characteristics of the Coloured Progressive Matrices (CPM) such as symmetry/asymmetry, vertical/horizontal, straight/curved lines, number of dimensions), or the algorithms (the rules or cognitive operations) required to solve them (e.g. Hornke & Habon, 1986; Carpenter, Just, & Shell, 1990). Raven, himself (Burke, 1958), categorised the items of the SPM according to five basic characteristics related to the algorithms required to solve the items – one for each of the five sets A to E as shown in Table 2.1.

The existing taxonomies were perceived to have disadvantages: firstly, it seemed several levels of analysis of items were needed: the existing taxonomies were either too broad or too detailed to be useful for this study; secondly, it seemed useful to combine the structure of an item and the (theoretical) processes or operations required to solve it; and, thirdly, some of the nomenclature used in previous research has not been strictly correct, (for example, "identity", in regard to a matrix, is not a repetition of identical figures in each cell of the matrix). Hence, a new taxonomy was developed to incorporate several levels of analysis, first, the Matrix size (1x1, 2x2, or 3x3); second, the five general Algorithms and (theoretical) processes associated with items; third, the Operators by which the algorithms are broken down to a more specific levels which describe detailed rules applying to the horizontal and vertical axes of a matrix, and the direction in which they operate; and, finally, Attributes which take account of the structural elements of the items the types and number of shapes and/or patterns making up the elements of a matrix. The taxonomy is shown in Table 2.2.

Most of the levels shown in Table 2.2 have sub-levels within them. The levels of major importance for this paper are the Algorithms and their sublevels and the Operators which detail the workings of the algorithms with regard to process and direction (horizontal or vertical) and, therefore,

Table 2.1. Raven's Description of the Five Types of SPM Items

SPM Set	Type
A	Continuous patterns
B	Analogies between pairs of figures
C	Progressive alteration of patterns
D	Permutations of figures
E	Resolution of figures into constituent parts

these are described in more detail in Tables 2.3 and 2.4, respectively. The algorithm Continuous has subcategories indicating the intra-matrix structure which may be "Jigsaw" (symmetrical pattern in the horizontal and vertical), or which may be similar to one of the major algorithms such as Reflection or Seriation. The algorithm Distributions has subcategories indicating whether the elements (or subelements) are rotating through the matrix or not, as does the algorithm Transformations. The algorithm Equation has subcategories indicating whether the equation is a union or an intersection and, within each of these, whether the elements (or subelements) are segregated or integrated.

The classification for each of the items of the SPM and APM (Sets I and II) at one level only (Algorithm) is shown in Table 2.5. The classification of the items was carried out by the researcher and a trained assistant in order to establish the reliability of the classification system and agreement was 100%.

In theory, respondents would have to use the operations associated with the algorithms in order to solve the items. Respondents may not always do this, for instance, a more perceptually-based, less abstract process may be used, but this would be considered a lower-order process which would be successful only with the easier items in a group of items using a particular algorithm.

Table 2.2. Hierarchical Taxonomy for the Raven's Progressive Matrices

Category 1 MATRIX SIZE	Category 2 ALGORITHM
1 x 1	Continuous ("jigsaw") (C)
2 x 2	Reflection (R)
3 x 3	Seriation (S)
	Distribution (D)
	Transformation (T)
	Equation (union or intersection) (EU or E)

Category 3 OPERATORS (on attributes)	Category 4 ATTRIBUTES
Same/different(across horizontal and vertical axes)	Whole/part elements
Operators, e.g. increasing number or size, rotation of elements	Shape e.g. square, cross
Direction e.g. left to right, top to bottom	Pattern, e.g. solid black, vertical stripes
	Number of elements or subelements

Table 2.3. Sublevels of the Category Algorithms in the Taxonomy for RPM Items

Algorithm	Subalgorithm
1. Continuous	a) "Jigsaw" b) Reflection c) Seriation d) Rotational distribution e) Equation
2. Reflection	No subcategories (differences occur at the level of Attributes)
3. Seriation	No subcategories (differences occur at the level of Attributes)
4. Distribution	a) Different elements – nonrotational Different elements – rotational b) Transformation of elements – nonrotational Transformation of elements – rotational
5. Equation	a) Union – segregated Union – integrated b) Intersection – segregated Intersection – integrated

Piagetian Stages

Although the formulation of Piaget's stages are well-known, to make the connections between psychometric and Piagetian theory in a later section, the characteristics of the stages are summarised here.

Piaget postulated that the development of cognitive functioning occurs in stages characterised by different modes of thinking or reasoning about and interpreting the world. Stages are universal, invariant and age-related.

Sensori-motor stage: This mode of interpreting the world is not addressed in this paper.

Pre-operational stage (Stage I): The child tends to interpret the world through perceived appearances rather than through inferred reality (Flavell, 1977), that is, s/he interprets the world in terms of what objects/situations look or sound like rather than in terms of an underlying reality: for a preoperational child, perceptual data *is* reality. Piaget used the term "figurative" to refer to such activities – activities which "represent reality as it appears, without seeking to transform it" (Piaget, 1970, p717).

Concrete-operational stage (Stages IIA and IIB): In contrast to the above, at this stage, a child interprets the world in terms of an "underlying

Table 2.4. Sublevels of the Category Operators in the Taxonomy for RPM Items

Operators *		Description
1. Increasing		number, size, amount or position
2. Form change		e.g. Three different shapes or patterns in a 3x3 matrix
3. Reflection		reflection of patterns and symmetrical/ asymmetrical figures
4. Rotation		rotation of element positions across rows/ columns of matrix
5. Transformation		sequential transformation of structure/quality of element the "2nd Col" rule: the 2nd column element gives the rule for transforming the 1st column
6. Union:	Segregated Integrated	overlap 1st and 2nd columns: retain all subelements
7. Intersection:	Segregated Integrated	a) overlapping elements cancel if similar (A ↔ B)' = C b) nonoverlapping elements remain (A ↔ B)" = C c) overlapping elements remain (A ↔ B) = C d) one element cancels others even when it is different
* Operators 1 and 4 are also coded according to the Direction of operation as follows:		
Horizontal plane:		Left to Right or Right to Left Top to Bottom or Bottom to Top
Vertical plane:		Clockwise or Anticlockwise

reality", or rule-governed ways of thinking (Howes, 1990). The child develops a logic of classes, relations and number, that is s/he reasons in terms of these objects, but cannot link any one object to any other except those that are adjacent to it (Piaget, 1972). The concrete-operational child can reverse actions, that is, realise that one action can reverse or nullify the opposite one, but is not yet able to link the two types of reversibility (negation and reciprocity). The child is limited by what is empirically given, that is, s/he is tied to reasoning about one particular situation at a time (hence the term "concrete" thinking).

Formal operational stage (Stages IIIA and IIIB): Adolescents at this stage differ from concrete thinkers in several ways. Firstly, they can

Table 2.5. All RPM Items in Increasing Order of Difficulty and Their Classification by the Taxonomic Category 'Algorithm'

Number	Difficulty (in logits)	Name	Algorithm	Number	Difficulty (in logits)	Name	Algorithm
1	-5.35	A3	C (J)	53	0.30	AdII/4	S
2	-5.01	A5	C (J)	54	0.30	E2	E
3	-4,52	B3	R	55	0.38	D10	D
4	-4.32	B2	R	56	0.52	AdII/11	E
5	-4.20	A6	C	57	0.54	E5	E
6	-4.13	A7	C	58	0.56	AdII/12	E
7	-3.86	A9	C	59	0.63	AdII/10	S
8	-3.50	D2	D	60	0.64	AdI/12	E
9	-3.45	AdI/4	C	61	0.65	B12	R
10	-3.40	B4	R	62	0.70	E4	E
11	-2.98	B5	R	63	0.76	AdII/14	S
12	-2.83	B6	R	64	0.78	C11	S
13	-2.79	AdI/1	C	65	1.08	AdII/16	E
14	-2.54	D5	D	66	1.09	AdII/17	T
15	-2.38	C7	S	67	1.15	AdII/13	S/D
16	-2.36	A10	C	68	1.29	AdII/15	T/E
17	-2.31	C3	S	69	1.32	AdI/9	S/T
18	-2.24	C2	S	70	1.36	AdII/8	D
19	-2.24	C5	S	71	1.37	E6	E
20	-2.24	B10	R	72	1.60	C8	S
21	-2.20	AdI/2	C	73	1.76	E7	E
22	-2.16	D3	D	74	1.81	AdII/19	E
23	-2.10	B11	R	75	1.84	AdI/11	T/E
24	-2.09	B9	R	76	1.94	AdII/20	E
25	-2.08	C9	S	77	2.17	AdII/22	E
26	-2.05	AdI/3	C	78	2.21	AdII/21	S/D/T
27	-1.86	B7	R	79	2.21	E8	E
28	-1.76	A8	C	80	2.29	D11	D/T
29	-1.71	B8	R	81	2.32	AdII/18	T
30	-1.73	D4	D	82	2.35	AdII/23	E
31	-1.67	C4	S	83	2.51	E10	E
32	-1.62	A11	C	84	2.55	D12	S/D/T
33	-1.61	D8	D	85	2.69	E9	E
34	-1.60	A12	C	86	2.96	AdII/26	D/S/T
35	-1.57	AdI/7	D	87	3.12	AdII/33	E
36	-1.42	AdI/5	S	88	3.15	AdII/30	T
37	-1.38	D6	D	89	3.26	AdII/31	S
38	-1.23	AdI/6	S	90	3.30	AdII/24	S
39	-1.22	C6	S	91	3.32	AdII/27	D/T

Table 2.5. All RPM Items in Increasing Order of Difficulty and Their Classification by the Taxonomic Category 'Algorithm' *(continued)*

Number	Difficulty (in logits)	Name	Algorithm	Number	Difficulty (in logits)	Name	Algorithm
40	-1.06	AdI/10	E	92	3.36	AdII/25	S
41	-1.05	D7	D	93	3.43	AdII/34	S/D/T
42	-1.04	AdII/5	S	94	3.71	C12	S
43	-0.93	AdII/6	S	95	3.83	E11	E
44	-0.77	AdI/8	S/D	96	3.84	AdII/28	S/D/T
45	-0.57	AdII/9	E	97	3.98	AdII/29	T
46	-0.53	AdII/1	D	98	4.01	AdII/35	E
47	-0.42	E3	E	99	4.01	E12	E
48	-0.41	D9	D	100	4.12	AdII/32	S/T
49	-0.41	AdII/3	S	101	5.86	AdII/36	E
50	-0.33	AdII/2	S				
51	-0.08	AdII/7	E				
52	0.01	C10	S				

conceive of not only the real and actual, but the possible, that is, they can go beyond reality to consider all theoretically possible situations associated with a task. Secondly, they tend to solve problems using hypothetico-deductive reasoning (Piaget, 1972). Thirdly, in contrast to children's intra-propositional thinking, the formal operator can reason inter-propositionally: s/he can consider the logical relationships between propositions (relations amongst relations) rather than the factual relationship between a proposition and reality only. This kind of logic is based on the use of the combinatorial system (the sixteen binary propositions of logic) which is the basis of combinatorial and permutational analysis, and the INCR group (Identity, Negation, Reciprocity and Correlation) which requires the combination of operations (Piaget, 1972; Bond, 1980). This constitutes an ability to reason at a second, or higher order level. Piaget saw the development of these abilities as relatively continuous from concrete to formal operations, but thought that development would be particularly rapid from about 12 years onward (Inhelder & Piaget, 1958).

All abilities available during each stage become consolidated during subsequent developmental stages, that is, concepts become more stable and robust as development proceeds.

Results and Discussion (Part II)

The joint scaling of the combined Matrices items and Piagetian tasks has already been shown graphically in Figure 2.1. The transition points and their significance have also been addressed in Part 1, as has the ordering of the Piagetian tasks. We now discuss, therefore, the ordering of the RPM items according to the taxomony provided; the relationships between the processing used in these items and the processing characteristics of the Piagetian stages; and, finally, the relationships between the RPM items and the transition points between Piagetian stages.

Ordering of the RPM Items

With respect to *the taxonomy of items*, although items from different taxonomic categories exhibit a range of difficulties, thereby resulting in the categories overlapping in location on the continuum, in general, there is a distinct sequence in the occurrence of certain algorithms at particular locations on the continuum in the order of difficulty as expected from Raven's original conception of the order and the taxonomy used here, that is, Continuous ("jigsaw" and other), Reflection, Seriation, Distribution (with and without rotations and/or Transformations), Equations (unions), and Equations (intersections). From a study of the order of the items both within and between these algorithms, it would be possible to identify aspects of processing which make an item more or less difficult to solve. The paper does not address this issue systematically and specifically, however, one aspect that is particularly relevant here is the number of algorithms involved in solving the items: the easiest items employ only one algorithm, and items employing two and then three algorithms are increasingly difficult. But there are very difficult items which employ only one algorithm, thus difficulty of items is related to either the type of algorithm or the number of algorithms, or both the type and the number.

Order of RPM Algorithms and Piagetian Tasks

The ordering of the algorithms for the items clustered around the transition points for all tasks is consistent in that the easier algorithms are associated with the earlier transition points and the algorithms become progressively more difficult at each succeeding transition point. This, again, supports the convergence of the two approaches to intellectual functioning: at a general level, one reflects the other. The ability to cope with items that employ two algorithms increases gradually through the stage of concrete operations, but the ability to deal with items using more than two algorithms is available only once formal operations are firmly established – almost at the transition between the substages IIIA and IIIB.

Based on these results, the answer as to whether the successful solution of matrix problems requires the attainment of formal operations or simply concrete operations, is, therefore, that it depends what the matrix problems are: concrete operational thinking are sufficient for the solution of matrix problems where the algorithm is fairly simple (one of two algorithms – Continuous or Reflection), or a easy example of a more difficult algorithm (Seriation or non-rotational Distribution with no transformations) which involve not more than two algorithms, both of which have to be relatively simple. During the stage of concrete operations, the ability to use operations simultaneously is quite limited. This supports Hubbs-Tait's conclusion that matrix items involving the interaction between variables require formal operational thinking (Hubbs-Tait, 1986).

Characteristics of RPM Items at Transition Points

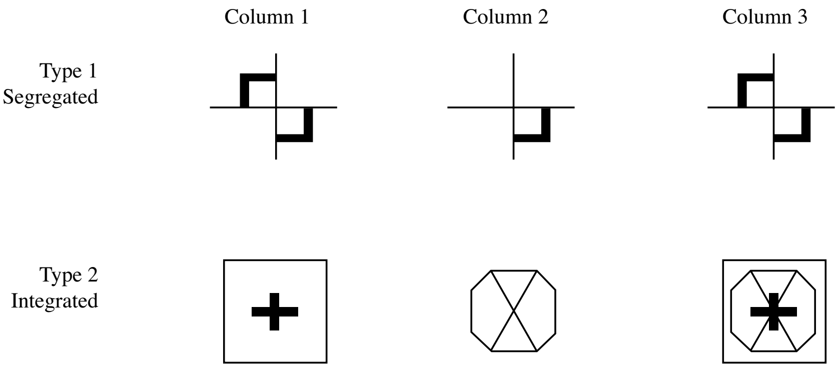
Table 2.6 shows the RPM items that occur within two standard errors above and below each of the thresholds between stages and sub-stages for each of the Piagetian tasks and the type of algorithm associated with each item according to the taxonomy presented in Table 2.3. It is evident that items occurring in the vicinity of each of the thresholds are characterised not only by particular algorithms but, further, usually include the first instances (in order of difficulty) of these algorithms. The four transition points are now addressed in turn.

First, considering the RPM items between the two major stages, II and III, the outstanding group consists of items which use the E algorithm (unions and intersections), in particular, for the first time, those items described as integrated unions where elements are superimposed directly on one another by joining the first and second columns (or rows) to form the third column (or row). No instances of E (intersection) occur before this transition. Three instances of the E (union) items occur before this point in the scale, they are, however, the first of the segregated unions: much simpler, than the integrated unions at a finer level of analysis in that their construction is such that joining the first two elements of the matrix results in a third element in which the subelements of the first two elements are still distinct from one another rather than overlapping one another completely to form a figure that looks very different from

Table 2.6. RPM Items (in Order of Difficulty) Occurring at the Thresholds Between Piagetian Stages, and Their Taxonomic Classification by Algorithm

Stage	Piagetian task	Location (in logits) +2 std errors	Continuous	Reflection	Algorithm Seriation	Classification Distribution	Transformation	Equation U	Equation I
I	Chem								
	Bal								
	Corr								
IIA/ IIB	Bal	-5.01 + 1.00	A3,A5,A6,A7	B3, B2					
	Corr	-3.03 + 0.76	Ad1/4,Ad1/1, A10	B4, B5, B6	C7, C3	D2, D5			
	Chem	-1.53 + 0.50	A8, A11, A12	B7, B8	C4, Ad1/5, Ad1/6, C6	D4, D8, Ad1/7, D6			
IIB/ IIIA	Corr	0.12 + 0.34			C10, Ad2/4	D10		Ad2/7, E2	
	Chem	0.30 + 0.34			C10, Ad2/4 Ad2/10	D10		E2, Ad2/11	E5, Ad2/12
	Bal	0.35 + 0.34		B12	Ad2/4, Ad2/10	D10		E2, Ad2/11	E5, Ad2/12, Ad1/12
IIIA/ IIIB	Chem	4.07 + 0.40			C12		Ad2/28		E11,Ad2/35, E12, Ad2/32
	Bal	4.58 + 0.52					Ad2/29		AD2/35, E12AD2/32
	Corr	7.14							

Figure 2.3. **Examples of One Horizontal Line of the Matrices of the Two Types of the Algorithm Union – Segregated and Integrated**



its parts. (In fact, the subelements are reflections of one another and the ability to operate with reflections is one that occurs earlier at the beginning of stage II, concrete operations.) Examples of the differences between Union (segregated) and Union (integrated) figures are shown in Figure 2.3.

The main difference between the Union (integrated) and Intersection (integrated) items and all earlier ones is that the former are the first items that require the respondent not only to go beyond the material presented in that they constitute *equations* involving the elements of the matrix, but also require a respondent to *know this explicitly*.

On the other hand, with items of the Union (segregated) type, it is possible to arrive at the correct answer simply because it "looks right", that is, one can rely on what Piaget described as a "figurative" way of thinking (producing what is given rather than having to transform the information in some way – the "logico-mathematical" way of thinking).

The joining (or separation) of the first two elements of the stem matrix in order to obtain the third element seems to be a second-order process which is not inherent in the perceptual properties of the stem matrix as is, for example, the redistribution of components across the elements of the stem matrix. This interpretation is consistent with Piaget's description of the characteristics of formal thinking as including an ability to deal with second-order relationships and more abstract ways of thinking and not being tied by what is immediately known or expected from the given perceptual task characteristics.

Second, *the transition from IIA to IIB for Chem and Corr* involves, in addition to some of the more difficult Continuous and Reflection items, the solution of Seriation (S) and Distribution (D) items for the first time. The IIA/IIB transition also involves items which employ two algorithms for the first time. Seriation and Distribution items involve either seriation or some of the possible combinations or permutations of elements making up the matrix. Further, combinations are classifications of all possible classifications and permutations are seriations of all possible seriations – they are operations on operations (Piaget, 1970). Thus, the fact that the IIA/IIB transition items for Chem and Corr involve the Seriation (S) and Distribution (D) algorithms for the first time fits well with the expected ability to begin to use elementary combinations which is a hallmark of these Piagetian tasks at this level.

The ability to solve D items is consolidated in the formal operational stage, but that, at the same time, the ability to deal with transformations (in association with rotations) develops. Transformations items are of the type where one figure is transformed in some way across the elements of the matrix (rather than the elements being comprised of three distinctly different shapes).

Note, too, that the Seriation algorithm is consolidated across concrete to formal operations, but that the major novel Seriation item type that occurs for the first time at the formal operational stage is one in which *position* (rather than number, size or amount) is the characteristic of the subelement that is seried, so that subelements appear to be moving across (or down) the matrix.

Third, the same *transition point (Stage IIA to Stage IIB) for the Bal task* is associated with a focus on R items (reflection): there is a clear similarity here between the operations involved in reflection and the ability to coordinate what is happening on both arms of the balance in the Balance task (negation and reciprocity). This is elaborated during concrete operational stage and is complete before the advent of formal operations. An interesting exception to this is the occurrence of B12 at the formal operations level Bal task). The interpretation here is that B12 has been misclassified – although it uses reflection, it is possible that it is a forerunner of an Intersection algorithm item in that there are two subelements, each of which is present or absent alternately in either the horizontal or the vertical. It is hypothesised that some of the processing abilities required for dealing with intersections are needed to be successful with this item.

Fourth, and lastly, the *transition from IIIA to IIIB is considered for Chem and Bal* only since no items occur in the vicinity of the corresponding Corr threshold which occurs well beyond the most difficult item of the

RPM. The main group of items here is comprised of intersection items where only non-overlapping elements between columns 1 and 2 remain in column 3 (or row 3). In particular, Intersection where two *different* patterns (or shapes) may eliminate each other, rather than the integration of two similar shapes resulting in their elimination . Again, this seems to fit well with the extension to being able to consider all possibilities and to coordinate several different operations simultaneously, which are hallmarks of the formal operational stage.

Another group of items occurring at this transition point is Transformations where the respondent has to recognise more than one type of transformation *and use them simultaneously*. In addition, other algorithms are involved. For these items, it is not possible for a respondent to regard transformations on elements simply as totally different figures (as it is in earlier Transformation items) – the types of transformation have to be recognised as such and coordinated with each other together with the use of multiple algorithms.

The ability to recognise the use of the same figure in different forms, to see the dependence amongst them even when more than one transformation is used within one figure, and take account of other operations at the same time, fits well with the notion of an intellect whose horizons are expanding to include all possibilities and which can deal with many possibilities at one time (again, an example of being able to perform operations on operations).

Another characteristic is that items which employ three algorithms start to occur just before this transition point.

No items occur in the vicinity of this transition point for Corr, however, it would be possible to predict what kind of items might do so, given their locations and what is known about the processes required at these levels in the Piagetian tasks.

Conclusions to Part II

The results of this study indicate the close relationship between Piagetian changes in modes of thinking and the processes associated with the solution of different types of psychometric items.

They also support the Piagetian notion of a major change in quality of thinking from concrete to formal operations and indicate that this major change is reflected, too, in the ability to solve psychometric items requiring particular types of processing. Changes involve the ability to recognise multiple aspects of a problem and to coordinate them, and to function less in a figurative (literal) and more in a logico-mathematical (abstract) way. This, in turn, strengthens the view that quantitative and qualitative development are intimately related to one another rather than being distinctly different from one another: they may be considered as being at different levels of scale, that is, the psychometric conception is simply a finer level of scale than the Piagetian one. It does not follow, however, that quantitative measures always at a finer level than qualitative – both may be found at all levels of scale: they are inseparable.

A further question that can be investigated through the use of this data is the relationship between the Piagetian transitions and intellectual growth spurts as measured on a psychometric variable. A major growth spurt, common to both sexes, has been demonstrated to occur during puberty, beginning for some children at the age of 11 years, and being completed by virtually all children by age 15 years (Andrich & Styles, 1994). This is entirely in accord with Piaget's postulation of a major qualitative change in reasoning from concrete to formal operations within a similar age range.

Overall, the conclusion is that the integration of the psychometric and cognitive-developmental approaches to the study of intelligence allows a deeper understanding of the variables under investigation and their relationships to one another.

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